Victorian Essential Learning Standards

Discipline-based Learning Strand

MATHEMATICS

Victoria
The Place To Be

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Contents

Discipline-based Learning

The domains within the Discipline-based Learning strand form a body of knowledge with associated ways of seeing the world and distinct methods of exploring, imagining and constructing that world.

Broadly in line with academic literature and consistent with practice in many schools, the *Victorian Essential Learning Standards* (VELS) identify the Arts, the Humanities, English and Languages Other Than English, Mathematics and Science as the disciplines for the curriculum over the stages of learning from Prep to Year 10.

Within the Discipline-based Learning strand the learning domains are:

The Arts

English

Humanities (Economics, Geography, History)

Languages Other Than English (LOTE)

Mathematics

Science

Students who develop a deep understanding of the concepts contained in the discipline-based domains are able to apply their knowledge in many different ways. The degree to which they are able to transfer their knowledge depends largely on the degree to which students have achieved mastery over Physical, Personal and Social and Interdisciplinary learning.

Research suggests that students develop deeper understanding of discipline-based concepts when they are encouraged to reflect on their learning, take personal responsibility for it and relate it to their own world. These approaches are explicitly defined in the Physical, Personal and Social Learning domains such as physical education and personal learning.

Students are better able to develop, demonstrate and use discipline-based knowledge and skills when they are able to employ knowledge in Interdisciplinary Learning such as Communication; Thinking; Information and Communications Technology; and Design, Creativity and Technology.

Mathematics

Introduction

Mathematics is a human endeavour that has developed by practice and theory from the dawn of civilisation to the present day. Many societies and cultures have contributed to the growth of mathematics often in times of scientific, technological, artistic and philosophical change and development. Complementary to this broad perspective of mathematics are the various mathematical practices that take place day to day in communities around the world.

While the usefulness of mathematics for modelling and problem solving is well known, mathematics also has a fundamental role in enabling cultural, social and technological advances, and empowering individuals as critical citizens in contemporary society and for the future. Number, space and measurement, and chance and data are natural and common aspects of most people's mathematical experience in everyday personal, study and work situations. Equally important are the essential roles that mathematical structure and working mathematically play in how we understand natural and human worlds.

Mathematics can be described in terms of its objects, what they are and how they came to be; its established body of knowledge and why this is held to be true; its effective application in science, technology and other domains; and the practice and activities of mathematicians past and present. Aims for essential learning in school mathematics are for students to:

- demonstrate useful mathematical and numeracy skills for successful general employment and functioning in society
- solve practical problems with mathematics, especially industry and work-based problems
- develop specialist knowledge in mathematics that provides for further study in the discipline
- see mathematical connections and be able to apply mathematical concepts, skills and processes in posing and solving mathematical problems
- be confident in one's personal knowledge of mathematics, to feel able to apply it, and to feel able to acquire new knowledge and skills when needed
- be empowered through knowledge of mathematics as a numerate citizen, able to apply this knowledge critically in societal and political contexts
- develop understanding of the role of mathematics in life, society and work; the role of mathematics in history; and mathematics as a discipline – its big ideas, history, aesthetics and philosophy.

Mathematical knowledge includes knowledge of concepts, objects, definitions and structures. A small collection of mathematical ideas, objects, structures, and relationships between these, is taken as undefined and given in a context. New mathematical objects, structures and relationships are developed from these moving from simple to more complex and sophisticated ideas and practices. The motivation for accepting certain things as given building blocks for mathematical knowledge may be initially related to intuitive understanding of particular ideas and objects experienced with respect to the natural or human worlds. These and their subsequent developments are not empirical knowledge, but abstract mathematical entities.

Whether mathematical knowledge is viewed as being essentially mind dependent or mind independent, discovered or constructed, its abstract nature gives rise to the applicability of mathematics in a wide range of contexts, as mathematical objects, structures and relationships do not depend on a particular context for their existence, but are interpreted to model key features of these contexts. This abstraction poses a challenge to the teacher and student alike, and both will need to draw on knowledge of the world and link this to mathematical knowledge and its application in various situations.

Mathematical reasoning and thinking underpins all aspects of school mathematics, including problem posing, problem solving, investigation and modelling. It encompasses the development of algorithms for computation, formulation of problems, making and testing conjectures, and the development of abstractions for further investigation.

Computation and proof are essential and complementary aspects of mathematics that enable students to develop thinking skills directed toward explaining, understanding and using mathematical concepts, structures and objects. They provide a framework for the development of mathematical skills and techniques exemplified in the use of algorithms for computation and for the development of general case arguments.

Dimensions

This domain has five dimensions. Standards for *Structure* are introduced from Level 3.

Number

Number provides for our sense of counting, magnitude and order. The natural (counting) numbers with zero extend to positive and negative signed whole numbers (integers) and through part-whole relations and proportions of whole numbers to the rational numbers (fractions and finite decimals or infinite recurring decimals).

Definitions of underlined terms are provided in the Glossary (page 41)

Proportions of lengths involving sides and/or diagonals of right-angled triangles and rectangles and arcs of a circle lead to the introduction of certain irrational real numbers such as the square root of 2, $\sqrt{2}$, the golden ratio *phi,* φ and fractions or multiples of *pi,* π.

Principal operations for computation with number include various algorithms for addition (aggregation), subtraction (disaggregation) and the related operations of multiplication, division and exponentiation carried out mentally, by hand using written algorithms and using calculators, spreadsheets or other numeric processors for calculation.

Space

Space provides for our sense of shape and location. These are connected through forms of representation of two- and three-dimensional objects and the ways in which the shapes of these objects and their ideal representations can be moved or combined through transformations. Key spatial concepts include continuity, edge, surface, region, boundary, connectedness, symmetry, invariance, congruence and similarity.

Principal operations for computation with space include identification and representation, construction and transformation, by hand, using drawing instruments, and also by using dynamic geometry technology.

Measurement, chance and data

Measurement, chance and data provides for our sense of unit, measure and error, chance and likelihood and inference. Measure is based on the notion of unit (*informal, formal* and *standard*) and relates number and natural language to measuring characteristics or attributes of objects and/or events. Various technologies are used to measure, and all measurement involves error.

Important common measures relate to money, length, mass, time and temperature, while probability is a measure of the chance or likelihood of an event. Other measures include area, volume and capacity, weight, angle, and derived rates such as density, concentration and speed.

Principle operations for computation with measurement include the use of formulas for evaluating measures, the use of technology such as data-loggers for direct and indirect measurement and related technologies for the subsequent analysis of data, and estimation of measures using comparison with prior knowledge and experience, and spatial and numerical manipulations.

Definitions of underlined terms are provided in the Glossary (page 41)

Structure

Structure provides for our sense of set, logic, function and algebra. It is fundamental to the concise and precise nature of mathematics and the generality of its results. Key elements of mathematical structure found in each of the dimensions of Mathematics are membership, operation, closure, identity, inverse, and the commutative, associative and distributive properties as well as other notions such as recursion.

While each of these can be considered in its own right, it is in their natural combination as applied to elements of number, space, function, algebra and logic with their characteristic operations that they give rise to the mathematical systems and structures that are embodied in each of these dimensions.

Principle operations for computation with structure include mental, by hand and technology-assisted calculation and symbolic manipulation, by calculators, spreadsheets or computer algebra systems, with sets, logic, functions and algebra.

Working mathematically

Working mathematically provides for our sense of mathematical inquiry: problem-posing and problem solving, modelling and investigation. It involves the application of principled reasoning in mathematics, in natural and symbolic language, through the mathematical processes of conjecture, formulation, solution and communication; and also engages the aesthetic aspects of mathematics.

In this dimension the nature, purpose and scope of individual work is connected to that of the broader mathematical community, and the historical heritage of mathematics through the discourse of working mathematically. Mental, by hand and technology-assisted methods provide complementary approaches to working mathematically.

Relationships between the dimensions

Number is related to the other dimensions through the aspects of counting, magnitude and order. It has logical and natural connections with *Measurement, chance and data*, and *Space*.

Number systems provide the basis for the development of algebraic relationships in *Structure* and the contexts and explorations used in *Working mathematically*.

Space is related to the *Number* and *Measurement, chance and data* dimensions through the aspects of shape and location. The properties of patterns, transformations, and symmetry provide links to *Structure* and *Working mathematically*.

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Measurement, chance and data is related to the *Number* and *Space* dimensions through the aspects of units, error, approximation, likelihood, angle, and the properties of two- and three-dimensional shapes. The application of measurement formulas and functions provide a link to *Structure*. A varied collection of practical contexts for generating and testing conjectures provides links to *Working mathematically*.

Structure is related to the *Number*, *Space* and *Measurement, chance and data* dimensions through the use of algorithms, patterns and functions. It is linked to *Working mathematically* through the key elements of mathematical language, concepts and relationships used in modelling and investigations.

Working mathematically is related to the *Number, Space and Measurement, chance and data* dimensions through the exploration of algorithms, patterns and functions, shapes and dimensions. It provides the processes for the development of inferences and deductions and for the exploration and proof of conjectures related to the *Structure* dimension.

Level 1

Learning focus

In *Number* and *Space*, students manipulate concrete and visual models to develop understanding of the fundamental mathematical concepts and objects of number, numeral, shape and location. They relate counting of discrete objects in sets to spatial patterns and arrangements of 1 to 20 objects with physical, visual and written representations including numerals. They apply number to establish sequence and order with respect to the elements of sets and model addition and subtraction by grouping together or by moving apart elements of sets.

They manipulate everyday objects to identify and describe the features of common two- and three-dimensional shapes that correspond to the spatial concepts of point, line, boundary, face, interior and exterior. They follow simple instructions for the location of objects and movement from one place to another in familiar situations. Students learn fundamental concepts related to *Measurement, chance and data* in situations where they need to measure and compare length, capacity, mass, time and temperature using descriptive terms such as *hot* or *fuller than* and/or by counting of informal units such as the length of a row of paperclips. They learn to make and check rough estimates of quantitative measurements. Students begin to recognise unpredictability and uncertainty in chance events such as a game of 'Snakes and Ladders' and identify and gather data required for a birthday party.

Students learn about fundamental aspects of *Structure* and *Working mathematically* by matching elements of different sets according to given instructions, such as one-to-one correspondence in a simple card game of memory, or a many-to-one correspondence between the students in a class and the first letter of their name. They explore patterns in number and space by manipulating objects according to simple rules and test the truth or otherwise of simple conjectures with respect to number, shape, pattern, measurement and data, simple time structures and the sequence of daily events. Students work with calculators to check the results of simple addition and subtraction and use drawing tools and geometry software to create and colour simple two-dimensional geometric shapes and visual patterns and composite objects based on these shapes.

In learning activities at Level 1, *Structure* provides notions of set, logic, function and algebra fundamental to the development of mathematical concepts, skills and processes in *Number*, *Space*, *Measurement, chance and data* and *Working mathematically.* The related standards for *Structure* are embedded across these dimensions to underpin an integrated approach to student learning.

Definitions of underlined terms are provided in the Glossary (page 41)

Standards

Number

At Level 1 students construct small sets of objects and elements according to simple descriptions and form correspondences between these sets based on simple relationships.

They use one-to-one correspondence to identify when two sets are equal in size or when one set is larger than another set or smaller than another set. They form collections of sets of equal size.

Students place sets in sequence of increasing size and use the numbers 0 to 20 to count and to determine the size of a given set, including zero for the empty set. They describe the position of an element in an ordered set using ordinal numbers up to ten.

They use materials to model addition and subtraction by the aggregation (grouping together) and dis-aggregation (moving apart) of elements in sets. They add and subtract by counting forward and backward using the natural numbers from 0 to 20.

Space

At Level 1 students recognise, copy and draw points, lines and simple free-hand curves and identify interior and exterior, edges; basic two-dimensional shapes such as triangles, circles and squares and basic three-dimensional solids and objects such as boxes and balls. They use attributes of shapes to construct small sets of geometric objects according to simple descriptions and form correspondences between these sets based on simple relationships.

They place and orientate shapes according to simple descriptions of relative location such as next to, beside, in front of, behind, over, under, and give and follow simple directions for locating an object and for movement from one place to another over a short distance.

They develop and follow simple instructions to move and place shapes and objects in familiar situations in relation to what they can see, and to move themselves from one place to another.

Measurement, chance and data

At Level 1 students measure and compare length, area, capacity and mass in relation to various familiar objects that are seen and handled using descriptive terms and/or informal units such as the length of a line segment using steps or paces, simple area covered such as a shape by two handprints, the capacity of containers such as half a glass of water, the weight of common objects such as a heavy schoolbag and duration such as the number of days until a birthday.

They recognise the flow and continuity of time and the use of natural cycles such as day/night, the seasons, and informal units such as heartbeats and hand claps at regular intervals to segment and describe the passage of time.

Students recognise and respond to unpredictability and variability in events, such as getting or not getting a certain number on the roll of a die in a game.

They identify and describe the outcomes of simple chance events such as the toss of a coin, and collect and display these using simple pictograph data related to their own activities which may include or events such as a birthday party.

Working mathematically

At Level 1 students make and test simple conjectures such as 'the larger an object the heavier it is', 'it is likely to rain after school today' and 'nine is four more than five'. They make rough estimates and check their work with respect to computations and constructions in *Number, Space, Measurement, chance and data* and *Structure*.

Students devise and follow ways of recording computations involving the use of materials, mental calculations and the digit keys and +, – and = keys on a four function calculator. Students use drawing tools such as simple shape templates and geometry software to draw points, lines, shapes and simple patterns and to copy a picture of a simple composite shape such as a child's sketch of a house using these shapes.

In this domain, standards for the *Structure* dimension are introduced at Level 3.

Level 2

Learning focus

In *Number* and *Space* students work with arrays of objects and base-10 models (units, longs, flats and cubes) to identify, order and model the counting numbers up to 1000. By using these materials they develop understanding of patterns in the numbers sequence mentally, by hand and using calculators (constant addition facility) to skip count up to 100 and *count on* and *count back*. They solve simple addition and subtraction using natural numbers to 100. They use groups of like materials to develop the notion of multiplication as repeated addition, and division as repeated subtraction of a set into equal-sized groups and represent these as rectangular arrays. They use regular geometric objects divided into equal segments and sets partitioned into arrays of their elements. To develop the concept of simple fraction and common fractions as parts of a whole unit. Students investigate the characteristics of simple shapes and solids with respect to similarity, symmetry and the application of simple transformations. They learn to devise and follow instructions in the forms of oral directions, informal maps and diagrams to locate a range of items or create routes to various places in and around their local environment.

As students increase the variety and range of objects and events used in *Measurement, chance and data* activities, they use informal units to measure length, area and volume, and start to recognise the importance of formal units for consistency in measurement. They recognise time patterns and cycles (second, minute, hour, day, sunrise and sunset, weekend and week) and the features of the calendar such as days of the week, date and month in practical situations related to their everyday family and school life, including telling the time using analogue and digital clocks. They learn to identify and collect data to answer posed questions, and then use simple graphical displays such as bar graphs and pictographs to organise and present the data. Students learn to recognise variability in chance events and describe qualitatively the likelihood and relative likelihood of everyday events using terms such as *unlikely* and *almost certain*, and *more likely* or *less likely*.

Students learn about *Structure* and *Working mathematically* by creating and manipulating sets of numbers, shapes, objects and patterns according to given criteria and they use a combination of every-day language and mathematical statements involving numerals, operations, connectives and relations to describe their mathematical working and results. Students test the truth or otherwise of conjectures by attempting to find examples or counter-examples and exploring special cases. They develop and consolidate their understanding of the commutative and associative properties for addition and multiplication. They model number, patterns, motion and data found in stories, familiar settings and daily activities using physical materials, diagrams and maps. During these activities students carry out well defined sequences of steps such as,

Definitions of underlined terms are provided in the Glossary (page 41)

using blocks, to complete a pattern in a design context, following a recipe, or preparing for school. Students use four-function calculators to check related computations and solutions to simple number sentences and equations and the accuracy of approximations to these computations.

Structure provides for the further development of key constructs of set, logic, function and algebra that are fundamental to the development of mathematical concepts, skills and processes in *Number*, *Space*, *Measurement, chance and data* and *Working mathematically.* The related standards for *Structure* are embedded across these dimensions to underpin an integrated approach to student learning.

Standards

Number

At Level 2 students partition a collection (the universal set) into distinct subsets according to simple criteria, and recognise when one set is a subset of another set. They construct and use sets of size 1, 10 and 100 to model place value and order natural numbers from 0 to 1000. Students describe simple fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ in terms of equal sized parts of a whole object, such as $\frac{1}{4}$ of a pizza (part-whole relationship), and collections such as $\frac{1}{2}$ of a set of 20 coloured pencils (subset-set relationship).

They use linear and rectangular arrays of like objects, and natural numbers, to skip count by 2s, 4s and 5s from zero to one hundred and to count to 1000 by 1s, 10s and 100s starting from any natural number. Students add and subtract one- and two-digit numbers by counting on and counting back. They mentally compute simple addition and subtraction calculations involving one- or two-digit natural numbers using number facts such as complement to 10, doubles and near doubles.

Students describe and calculate simple multiplication as repeated addition such as $3 \times 5 = 5 + 5 + 5$; and division as repeated subtraction, such as 8 divided between 4, and as partitioning of a set into equal-sized subsets. They use commutative and associative properties of addition and multiplication in mental computation.

Space

At Level 2 students recognise lines, surfaces and planes, corners and boundaries; familiar two-dimensional shapes including rectangles, rhombuses and hexagons, and three-dimensional shapes and objects including pyramids, cones, and cylinders. They partition a collection of geometric shapes, such as a set of attribute blocks, into distinct subsets according to simple criteria, and recognise when one set of shapes is a subset of another set of shapes.

They recognise and describe symmetry, asymmetry, and congruence in these shapes and objects. They accurately draw simple two-dimensional shapes by hand and construct, copy and combine these shapes using drawing tools and geometry software.

Students apply simple transformations to shapes (flips, turns, slides and enlargements) and depict both original and transformed shape together.

They specify location as a relative position, including left and right, and interpret simple networks, diagrams and maps involving a small number of points, objects or locations.

Measurement, chance and data

At Level 2 students use physical models, money and diagrams to illustrate ordered and unordered sets of numbers, shapes, objects and data and carry out related computations and manipulations. Students make, describe and compare measurements, characteristics or outcomes such as length, width and height, mass, certain, likely, unlikely and impossible.

They develop and apply criteria for the use of informal units, including non-uniform measures, such as hand-span, and uniform measures, such as icy-pole sticks for length; use formal units such as hour for time or litre for capacity; and use the standard units for length (metre), mass (kilogram) and time (second). They describe temperature qualitatively and informally measure and compare areas as enclosed space or space covered. They judge relative capacity of familiar objects and containers by eye and make informal comparisons of weight by hefting.

Students recognise the key elements of the calendar and place in sequence days, weeks and months. They describe common and familiar time patterns and such as the time, duration and day of regular sport training, and tell the time to hours and half-hours using an analog clock, and to hours and minutes using a digital clock.

Students make predictions for the likelihood of outcomes of simple random and non-random events, using qualitative descriptors for more likely or less likely, such as such as whether 6s are harder to roll than 2s on a die. They collect simple categorical and numerical data (count of frequency) and present this data using pictographs and simple bar graphs and make predictions about the outcome of chance experiments in response to queries.

Working mathematically

At Level 2 students make and test simple conjectures by finding examples, counter-examples and special cases, and informally decide whether a conjecture is likely to be true in general.

Students use place value to enter and read displayed numbers on a calculator. They use a four-function calculator, including use of the constant addition function and × key, to check the accuracy of mental and written estimations and approximations and solutions to simple number sentences and equations.

In this domain, standards for the *Structure* dimension are introduced at Level 3.

Level 3

Learning focus

In *Number* students develop their understanding of number value and order from hundredths to tens of thousands. They routinely use multiples to skip count and create number patterns, including using multiples of 10, to explore more fully place value and the operation of multiplication. They work on practical problems in which the complexity of computations extends to include addition and subtraction of three-digit numbers, multiplication by single digits, and division by a single-digit number. Mental computations involve numbers up to 30. They learn to work with equivalent fractions and apply this to the addition and subtraction of simple common fractions.

In *Space* students explore the orientation of lines and classify the key features of two- and three-dimensional shapes and their representations. They develop and follow instructions for the creation of patterns based on simple tessellations and designs, and relate these to work in other domains and to solving practical problems in and around the home. During these activities students become familiar with the concept of angle as a measure of turn and, in conjunction with grid references and the cardinal compass points, use it to specify location with increased precision and work with maps that include a combination of informal and formal directions.

In *Measurement, chance and data* students work in contexts where they use informal (non-uniform and uniform), formal and standard measurement units to estimate and measure an increasing range of characteristics and attributes of objects and events. They learn to read scales and clocks with accuracy to the major divisions, and extract information from calendars, lists, tables and simple graphical displays when they plan activities and carry out tasks. They observe events in daily life such as recreational games and sports where they recognise natural variability in chance events and order these events from least likely to most likely. Students construct simple frequency graphs from experimental and collected data across the domains and in everyday life, and use simple Karnaugh maps (two-way tables) to display categorical data.

In *Structure* students investigate sets formed by the counting of equal-sized subsets of tens, hundreds and thousands to develop multiplicative thinking, and also partition sets into smaller equal-sized subsets to develop the concepts of division and remainder. Students investigate sets formed by mathematical properties, operations and classifications, for example, equivalent fractions, shape transformations, and outcomes of random events. They recognise and learn to use the commutative and associative properties to support computation, and use simple algorithms for computation. They analyse and

Definitions of underlined terms are provided in the Glossary (page 41)

classify shapes and solids in terms of their geometric properties. Students develop an appreciation of formal measurement units in length, area and time contexts, and the use of scales and conversions between units. They learn to identify and describe localities and features on local and larger-scale maps and can devise suitable routes between places.

Students generate simple number sequences using recursion such as 'the next term in the sequence is two more than the previous term'. They create and solve simple equations involving integers and simple fractions, and further develop their ability to use mathematical symbols and terminology, for example, decimal form, brackets, the division symbol and inequality symbol, and the logical connectives *and*, *or* and *not*.

In *Working mathematically* students learn to develop conjectures about the concepts of number, space, measurement and chance: the generality of patterns, numbers and shapes; the size and type of numbers resulting from computations; the effects of transformations of shapes; the outcomes of measurements and random experiments; and inferences from collected samples. Students learn to choose suitable representations from a variety of models that represent number, computations, patterns and shapes.

Students learn to recognise practical applications of mathematics in daily life, including shopping, travel and time of day, and investigate the historical development of some mathematical concepts and structures. Students use a range of technologies, including calculators, computer drawing packages and measuring tools, and learn to construct and describe simple algorithms using mathematical conventions.

Standards

Number

At Level 3 students use place value to determine the size and order of numbers from hundredths to tens of thousands. They round numbers up and down to the nearest unit, ten, hundred, or thousand. They compare and order simple common fractions such as $\frac{3}{4} > \frac{2}{3}$.

Students skip count forwards and backwards, from various starting points using multiples of 2, 3, 4, 5, 10 and 100. Students devise and use algorithms for whole number problems of addition and subtraction involving three-digit numbers; multiplication by single digits (based on automatic recall of multiplication tables) and multiples and powers of ten; and division by a single-digit divisor (based on inverse relations in multiplication tables). They devise and apply algorithms for the addition and subtraction of numbers to two decimal places. They add and subtract simple common fractions with the assistance of physical models.

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Students perform mental computations involving numbers up to 30 accurately and reliably. Numbers are estimated and ordered to two decimal places. They predict the accuracy of estimations for computation and recognise whether these are likely to be over-estimates or under-estimates.

Space

At Level 3 students recognise and describe the orientation of lines as vertical, horizontal and diagonal. They describe angle in terms of rotation of line segments which meet at a common end-point.

Students recognise polygons, prisms and pyramids and their component parts such as edges, vertices and faces. They use and interpret two-dimensional representations of three-dimensional objects or parts of these objects, for example, nets, cross-sections and simple projections. They describe what is seen and not seen of a simple object from different positions. They recognise and construct simple tessellations and follow instructions to produce geometric designs such as tangrams.

Students use and compare ways of locating and identifying places on maps and diagrams. They develop and test instructions to specify travel directions and location using compass directions, N, S, E and W, and grid references such as 'A5' on a street directory.

Measurement, chance and data

At Level 3 students extend the range of characteristics and attributes, estimated and measured, using informal and formal units to include angle (simple fractions of a complete turn), temperature and weight.

Students estimate and measure length, area, volume, mass and time using appropriate instruments. They recognise and use different units of measurement (informal, formal and standard metric measures) in appropriate contexts, and interpret linear and circular scales in familiar situations such as measuring weight. Students describe and interpret the numbers on analog clocks in relation to the minute and hour hands, and interpret timetables and calendars in relation to familiar events.

Students rate everyday outcomes in terms of likely occurrence and informally and qualitatively describe the *fairness* of events. They plan and conduct chance experiments with respect to natural variability and tally results of these experiments.

Students identify numerical data as discrete or continuous and construct column and bar graphs to display frequency data of ordinal categories.

Structure

At Level 3 students construct number collections using counting of composite sets of units such as 2, 3, 4, 5, 10 and 100. Students investigate and record sequences of decimal numbers generated using multiplication or division by 10. They partition sets into equal-sized subsets to carry out division and recognise that the sharing of a collection into equal-sized parts frequently leaves a remainder. Students identify the set of all possible outcomes of a simple chance event (the event space) and use Karnaugh maps to specify the possible combinations of two attributes.

Students recognise the importance and meaning of the '=' sign in mathematical statements and technology displays to indicate the result of a computation and to indicate equivalence. They use the commutative and associative properties in combination to facilitate computations such as $7 + 10 + 13 = 10 + 7 + 13 = 10 + 20$. They use the distributive property for multiplication over addition in simple computation.

Students classify and describe angles, polygons and solids according to their properties. They describe and summarise the effects of rotations, reflections, transformations and shadow projections on shapes with respect to what changes and what does not change (invariance).

Students identify variables and perform simple operations on variables. They construct and solve simple equations involving missing numbers and '='. They recognise samples as subsets of a set (the population under consideration). They organise data into lists and Karnaugh maps.

Working mathematically

At Level 3 students use brackets to give priority to an operation in a simple sequence of operations. Students follow and interpret algorithms and methods of approximation used by others. They formulate and test conjectures to investigate number (for example, the shapes that can be used to model common fractions); computations (for example, the nature of the product of even and/or odd numbers); number patterns (for example, the patterns of last digits produced by multiples of a given number); measurement (for example, the relationship between size and capacity of a container); and shapes (for example, the effects of reflections, slides/translations and rotations on the orientation of a shape).

Students describe and explain why some shapes tessellate, why some shapes have different forms of symmetry, and which solids have nets. Students represent depth in drawings and describe 'what is not seen' in three-dimensional drawings. They use and interpret physical models, the place-value model, and diagrams to explore the properties of numbers, shapes, and location, and to represent computations and measurements. Students develop and apply appropriate methods for collection and presentation of survey and simulation data.

Students apply number skills to everyday contexts such as shopping, with appropriate rounding to the nearest five cents. Students illustrate tiling patterns and stacking of solids. They identify familiar places and routes from local and regional maps and diagrams, and relate daily activities to clock times. Students describe uses of mathematics in earlier times with respect to different numeration systems and bases, place value and algorithms for computation.

Students use a calculator to check the accuracy of estimations and computations involving whole numbers and decimals to two places. They use a calculator to identify and classify the form of decimal values that result from division of natural numbers.

Students use computer software to create shapes, tessellations, maps and diagrams, and to organise and present data. They use a range of mechanical and electronic measuring instruments to support mathematical development at this level.

Level 4

Learning focus

In *Number* students work with the size and order of large and small numbers including negative numbers, and rational numbers in fraction and decimal form. They learn to identify natural numbers and their factors as prime, even or odd, and to use decimals, ratios and percentages to represent equivalent forms of common fractions.

Students develop and apply mental and written algorithms for the addition, subtraction, multiplication and division of natural numbers; the addition, subtraction, multiplication of decimals (to two decimal places); and the addition, multiplication and subtraction of common fractions. In division of natural numbers, they identify and interpret remainders as fractions and recognise the role that remainders play in developing algorithms for finding factors of numbers and decomposing numbers into their product of powers of prime numbers. They routinely make estimations and approximations in numerical computation and learn to make judgments about the appropriateness, effectiveness and reasonableness of these estimates and approximations.

In *Space* students develop their understanding of mathematical properties of shapes and solids and incorporate concepts of size and scale into their descriptions of these shapes and solids. They work with the concepts of boundary, finite and infinite, in relation to lines, surfaces and shapes, and use these ideas to investigate self-similarity in lines and shapes. Students learn to describe the features of shapes and solids that remain the same (are invariant) or change when the shape or solid is enlarged or reduced. They use the ideas of size, scale and direction to specify the relative positions of places and objects in maps and, in the process, apply concepts of shape and connectedness to represent and interpret simple networks.

In *Measurement, chance and data* students learn to use suitable instruments to measure accurately the characteristics of length, area, volume, capacity, angle, time and temperature in formal and standard units. They describe time elapsed in hours, minutes and seconds, and in simple decimal subdivisions of a second (tenths and hundredths). They measure the perimeter and surface area of a range of shapes and describe the accuracy and adequacy of their measurements in context. Students learn to distinguish between discrete and continuous measurement data and to apply measures of centre and simple measures of spread to informally describe simple characteristics of the distribution of data in a set. They refine their descriptions of chance (random) events from impossible to certain using mathematical words and fractions between 0 and 1, and follow simple simulations of chance events. They use mechanical and electronic technology to simulate outcomes in a random experiment, and a scientific calculator to calculate the mean of a discrete or

continuous data set. They also use technology to create graphs of data sets and find and extract relevant information from a database available on the Internet.

In *Structure* students learn to form and specify sets based on given properties or criteria (for example, sequences generated from small composite units for counting forwards and backwards); representation of natural numbers in different bases; shapes that enclose space in two dimensions and solids in three dimensions; common objects that may be used to make measurements in practical situations; and outcomes in chance experiments. They learn to classify patterns of remainders formed when larger natural numbers are divided by small natural numbers such as 3 or 5; and classify shapes and solids according to their geometric properties.

Students sort and classify lines in shapes and solids found in the environment according to their relative orientations, and identify shapes that have reflection symmetry and/or rotation symmetry. They learn to distinguish the amount of turn in angles relative to straight, right and zero angles, and find the shortest paths or routes between places on a map. They use the concept of one-to-one correspondence to construct number lines involving whole numbers, decimals to one decimal place and simple common fractions.

Students draw Venn diagrams and Karnaugh maps to help them test the validity of simple deductive arguments involving the quantifiers *none*, *some* and *all*. They make judgments about the appropriateness, scope and reasonableness of procedures for tasks, and develop simple algorithms involving words, diagrams and mathematical symbols. They generate sequences using recursion (computing the next term from the previous term or terms), and develop function rules for computing terms in sequences depending on their position in the sequence.

Students learn to use the identity elements for arithmetic operations on integers and rational numbers, and apply the relationship between identity and inverse to the addition, subtraction and multiplication of integers and fractions. They play and interpret games in which the rules are based on transformations of shapes, or combinations of transformations of shapes, and form a mathematical structure.

In *Working mathematically* using the processes of specialising, exemplifying, justifying and refuting, students learn to make judgments about the truth of conjectures regarding even and prime numbers, remainders, shapes and their properties, and relationships in measurement. They make and test conjectures and generalisations about sets of numbers and shapes and their properties, and develop convincing, principled arguments for propositions. They use concrete materials, diagrams and functions as models to test conjectures relating to the *Number*, *Space* and *Measurement chance and data* dimensions.

Definitions of underlined terms are provided in the Glossary (page 41)

Students develop, use and refine criteria for collecting, organising and presenting data. They identify situations in everyday life where estimates of numbers and computations are considered appropriate, and investigate the methods used to make these estimates. They apply concepts from *Number* and *Space* to develop strategies for games, and identify myths and misconceptions about chance and fairness in everyday situations. They identify key concepts and developments from contexts in the history of mathematics, for example, triangular, square, pentagonal, hexagonal and perfect numbers in classical Greek mathematics.

Students develop and use estimation procedures to predict and check the results of computations carried out with technology. They use technology for more complex and extended computations involving a series of operations with small and/or large numbers. They use application programs in a graphics calculator to explore and analyse games and puzzles involving numbers. They carry out a sequence of instructions to draw a shape, solid or net of a solid, and translate these instructions into a language or form that can be used in a computer software package or graphics calculator to represent shapes and solids under transformations.

Standards

Number

At Level 4 students comprehend the size and order of small and large numbers (from thousandths to millions), including negative numbers, common fractions and decimals. Students accurately estimate the size of fractions and decimals in the vicinity of 0 and 1 relative to 0, ½ and 1. They identify numbers and their factors as square, prime or composite, and interpret these numbers and their factors in terms of the area and the dimensions of their corresponding rectangular geometric arrays. They recognise and evaluate simple powers of natural numbers such as $2^4 = 16$.

Students explain and use mental and written algorithms for the addition, subtraction, multiplication and division of natural numbers; the addition, subtraction, multiplication of decimals (to two decimal places); and the addition, multiplication and subtraction of common fractions. They represent natural numbers in other bases. They construct and recognise multiples of integers (including lowest common multiple) and common fractions, and interpret constant multiples of a number as scale factors of the number. They use decimals, ratios and percentages to find equivalent representations of common fractions.

Students identify and interpret remainders as fractions and recognise the role that remainders play in algorithms for finding the factors of natural numbers. They use repeated division by increasing primes to express numbers as a product of powers of prime numbers, for example, $360 = 2^3 \times 3^2 \times 5^1$.

When using estimates of numbers in computation, students apply strategies appropriate for the situation, in particular, mental computations. They develop and use criteria for deciding if an estimate of a computation is reasonable or not.

Space

At Level 4 students identify the mathematical properties of horizontal, vertical, parallel and perpendicular lines in relation to each other; shapes and solids, including prisms, pyramids, cylinders and cones; and incorporate the ideas of angle, size and scale into descriptions of the features of these shapes and solids. They explain the idea of finiteness and non-finiteness in relation to lines and surfaces, and use recursion to investigate the idea of self-similarity of shapes. They make two-dimensional representations of three-dimensional objects.

Students use sketches of shapes and solids to represent the surrounding environment, and describe in mathematical language their relative sizes in that environment. Students also use the ideas of size, scale and direction when referring to the relative positions of places and objects in maps, and demonstrate understanding of shape and connectedness in diagrams of networks.

Students develop a sequence of instructions for drawing a shape, solid or net of a solid, and adjust these instructions to take account of scale. They formulate and test procedures, expressed in terms of compass points and simple coordinate systems, that describe how to get from one place to other places. They use and interpret conventional symbols and language in activities relating to place, direction, paths and scale in maps.

Measurement, chance and data

At Level 4 students accurately measure the characteristics of length (including perimeter), area (including surface area), volume, capacity, angle, time (including duration of time) and temperature in formal and standard units using appropriate instruments and scales. They choose accuracy of measurement relevant to the situation at hand and sufficient to distinguish between the sizes of things with the same characteristic, and find the corresponding difference between these sizes to that accuracy.

Students refine their descriptions of chance (random) events in the range from impossible to certain using words and fractions or decimals between 0 and 1. They explain the role of symmetry in chance situations and experiments involving equally likely events (for example, that the symmetry inherent in a device used to generate random events may be used to calculate the probability of the outcomes in each event). They comprehend that experimental estimates of probabilities (relative frequencies) converge to the theoretical probability values in the long run. They comprehend how chance events may be simulated (for example, randomly choosing a birth month by

selecting from a shuffled pack of cards without kings), and that simulations provide models (estimates) of situations that are impractical to deal with without using an empirical approach.

Students classify numerical data as discrete or continuous, and collect, organise, analyse, interpret and represent categorical, ordinal and numerical data in response to planned questions. They attend to the clarity of the questions, sampling techniques, and methods used to present data. They recognise and describe the relationship between measures of centrality and simple measures of spread used to describe and order data in a set. Students follow a plan and sequence of instructions involving shapes and measurements to construct an object from prefabricated parts, for example, a piece of furniture or a model car. They calculate and compare the times taken by various people to complete activities.

Students calculate probabilities associated with experiments involving equally likely events (for example, the probability of each outcome when two die are rolled), and the probabilities of symmetric events in the event space, and the mean, median and range for grouped and ungrouped data. They construct graphs to represent data sets and devise suitable scales (nominal, ordinal or interval) for the reference lines (axes) for the categories and frequencies of the data.

Structure

At Level 4 students form and specify sets of numbers, shapes, transformations and data according to given criteria and/or conditions such as equivalence, congruence and sampling for particular attributes. They identify the nature of the set (the population) and data from which samples are drawn as finite or infinite and discrete or continuous. They use Venn diagrams and Karnaugh maps to test the validity of simple deductive arguments involving simple applications of the quantifiers *none*, *some* or *all* to sets.

Students identify variables and related variables in everyday situations, and explain the ideas of change, dependency and allowable values in relationships between pairs of variables. They interpret sketch graphs involving functions of a single variable. They construct rules for sequences using recursion relations and relations that depend on the position of the term in the sequence. They describe the features of shapes or solids that remain the same or change when the shape or solid is enlarged or reduced.

They describe general patterns using words, numbers, diagrams and symbols. They establish equivalence of simple mathematical expressions involving properties, such as the distributive property for multiplication over addition, $a(b + c) = ab + ac$. They identify and apply the identity and inverse elements for the arithmetic operation on integers and rational numbers, and for simple transformations in space.

Working mathematically

At Level 4 students explain why a few successful examples are not sufficient to form a generalisation and how a single counter-example suffices to invalidate a generalisation. They make and test conjectures about the generalised forms of numbers in terms of divisors, factors and remainders; shapes and their properties and related measurements.

Students use appropriate physical models and graphs when testing the truth of conjectures. They design algorithms as models of mathematical processes such as the construction of an equilateral triangle. They engage in a planned investigation involving mathematical modelling and refine a model in terms of its formulation and interpretation. They identify the historical evolution of key mathematical ideas such as the emergence of negative numbers.

Students identify situations in everyday life where estimates of numbers and computations are considered appropriate, and investigate the methods used to make these estimates and estimate likelihood from simulations. They collect and analyse data about people's beliefs about fairness in games of chance.

Students use the memory function on a scientific or graphics calculator to do computations with a series of operations involving small and large numbers, and, with the aid of a scientific or graphics calculator, use estimation procedures to predict and check the results of computations. They use a scientific or graphics calculator to implement algorithms to find factors and prime factors of numbers and to explore facts and puzzles involving numbers. They use graphics calculators or computer-drawing packages and application programs to represent shapes and solids under a range of transformations, and use technology to generate simple simulations of events such as gender and order of children born in a family.

Level 5

Learning focus

In *Number* students learn to classify numbers encountered at earlier levels as natural numbers, integers and rational numbers. They express natural numbers as factors of powers of prime numbers, and evaluate simple expressions involving natural numbers, integers and simple rational numbers in base-exponent form, in particular as perfect squares. They comprehend and use ratio as a representation of relative size, and proportion as equivalent ratio, and learn to consider percentage as proportion relative to 100. Students develop understanding of the concept of constant rate of change in terms of constant ratio between two variables. They are introduced to *pi*, π, as the constant ratio of circumference to diameter in circles, rational approximations to *pi*, π, and applications of this number.

Students develop facility with written algorithms for arithmetic operations on integers and rational numbers, and use increased familiarity with number facts and properties to extend their capabilities with mental computation involving integers, finite decimals and fractions. They perform multiple-operation computations, including cases where brackets are used to specify order, using mental computation, written algorithms and technology as applicable. They develop facility with the general conventions for implied order of computation when technology is used. Students learn to apply operations of squaring a positive number and taking the square root of a positive number as inverse operations when calculating and estimating squares and square roots of positive numbers. Students use a range of methods to estimate numbers for use in computations, and note the effects of rounding down and rounding up approximations to the results of computations.

In *Space* students learn to use the concepts of similarity and congruence to interpret the relative size of shapes and forms in patterns and designs. They construct plane shapes, including regular polygons, and their component parts according to specifications. Students extend their explorations of tessellations to the shapes used in architecture and design, in particular with respect to depth and perspective in two-dimensional representation of three-dimensional objects. They learn to develop nets for common three-dimensional shapes and use these to construct corresponding geometric objects.

Students investigate and use, in a range of contexts, the mathematical properties of angles formed by intersecting straight lines. They interpret and use a range of familiar and common maps of locations from small to large scale, using plans and grids. They explore the patterns formed by following procedures involving simple transformations or movements around grids.

Definitions of underlined terms are provided in the Glossary (page 41)

They use networks to represent relationships in everyday life, for example, a tree diagram for a family tree and a network for the boundaries and lines on a playing field or sports court.

In *Measurement, chance and data* students measure length, perimeter, area, surface area, volume, capacity, angle in shapes and solids, time, and temperature. They use mensuration formulas to calculate the area and perimeter of circles, parallelograms, regular polygons, and the surface area and volume of prisms and calculate and use simple rates involving time and area. They learn to convert metric units into smaller or larger units as required, and make estimates of parts of scales on measurement instruments to improve the accuracy of their measurements. They make judgements about the reasonableness of the accuracy of measurements and related estimates and error.

Students develop an understanding of the relationship between long-run frequency and probability and estimate probabilities of events based on empirical data from experiments and simulations of one- and two-event trials, and describe related event spaces. They apply simple procedures for generating random numbers in simulations.

Students learn to distinguish between a sample and a population and also between a survey and a census and develop understanding that the purpose of sampling is to make inferences and predictions about the population from which the sample was taken with some measure of confidence. They apply by-hand procedures technology to organise and present discrete and continuous data and to obtain summary statistics that describe the data sets (mean, median, mode and range).

In *Structure* students draw together their experience with numbers by organising them into collections of natural, integer, rational or irrational numbers and use Venn diagrams and tree diagrams to represent the relationships between these sets of numbers. They give examples of the application of the commutative, associative and distributive properties in arithmetic, and provide counter-examples where these do not apply.

Students learn to identify the identity element and inverse of an arbitrary number for the integers and rational numbers for their respective operations of addition and multiplication and apply these to the re-arrangement of simple mensuration formulas and the demonstration of the equivalence between simple algebraic expressions. They become familiar with truth values for the connectives *and*, *or*, *not*, implication and equivalence, and the meaning of the quantifiers *none*, *some* and *all*. They apply these to the specification of sets defined in terms of one or two attributes using tree diagrams, Venn diagrams, and Karnaugh maps using complement, intersection and union.

Definitions of underlined terms are provided in the Glossary (page 41)

Students develop an understanding of the concept of function and the representation of functions by ordered pairs (tables of values), graphs and rules applied over a given domain. They work with linear functions and other simple functions of a single variable, both in exploring number patterns and as functional models for practical situations. They solve simple equations that are amenable to solution by a sequence of inverse operations and determine the truth values for mathematical expressions, including simple inequalities.

In *Working mathematically* students learn to move from consideration of multiple examples of true propositions to informal consideration of justification for general conjectures based on the examples. They develop simple mathematical models and procedures, with awareness of basic assumptions, and collect relevant data, represent relationships in mathematical terms, and make and test predictions from models. Students learn to work with a range of standard mathematical models in practical situations and apply routines and algorithms relevant to those situations. They consider interpolation and extrapolation from given data used to construct these models.

In the process, they learn to use technologies such as geometry software, graphics calculators, spreadsheets and computer algebra systems to assist in the development of mathematical ideas and carry out relevant computations.

Definitions of underlined terms are provided in the Glossary (page 41)

Standards

Number

At Level 5 students identify complete factor sets for large natural numbers and express these natural numbers as products of powers of primes. They express natural numbers base 10 in binary form, for example $42_{10} = 101010_2$, and add and multiply natural numbers in binary form. They express rational numbers as fractions and decimals (finite and infinite recurring) and are adept at ordering rational numbers expressed in decimal form. They know the decimal equivalents for $\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{8},\frac{1}{9}$ and use these to give decimal equivalents for rational numbers such as $\frac{4}{9}$ and $2\frac{3}{8}$. They evaluate natural numbers and simple fractions given in base-exponent form, for example, $5^4 = 625$ and $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$, and find rational square roots of rational numbers that are perfect squares, for example, $\sqrt{0.81} = 0.9$ and $\sqrt{\frac{25}{16}} = \frac{5}{4}$.

Students write sequences of equivalent fractions for a fraction given in simplest form, for example, $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \ldots$ They understand ratio as both *set*:*set* comparison (for example, the number of boys in a class: the number of girls in a class) and *subset*:*set* comparison (for example, the number of girls in a class: the number of students in the class), and find integer proportions of these. They also determine proportions as percentages where these can be obtained directly as equivalent fractions with denominator 100.

Students carry out arithmetic computations involving natural numbers, integers and finite decimals using mental and/or written algorithms with one- and two-digit divisors in the case of division. They use calculators for arithmetic computations involving several operations on natural or rational numbers of any size. They carry out exact arithmetic computations involving common fractions and use knowledge of perfect squares when calculating and estimating squares of numbers and square roots of numbers. They use scale multiples of *pi,* π, in measurement formulas related to circles.

They use a range of strategies for approximating the results of computations, such as front-end estimation, clustering around a common value, rounding, and seeking nearby numbers which are multiples of the divisor when doing division.

Space

At Level 5 students construct exactly two-dimensional and simple three-dimensional shapes according to specifications in terms of length, angle and adjacency. They relate similarity and congruence in the case of single transformations to enlargement from a common fixed point, and the superimposition of geometric objects respectively. They form patterns of shapes, including simple tessellations of a single shape, demonstrating an understanding of similarity and congruence. They use single-point perspective to make a two-dimensional representation of a simple three-dimensional object, and use two-dimensional nets to construct a simple three-dimensional object such as a prism or a platonic solid.

Students use the properties of lines, parallel lines, and transversals of these lines to calculate angles that are supplementary, corresponding, allied and alternate, and describe and apply the properties of regular and irregular polygons, in particular, triangles and quadrilaterals.

Students use coordinates and quadrants to identify positions and directions in the plane, and interpret and use lines, grids, contours, isobars, scales and bearings to specify location and direction on plans and maps. They also use networks, including tree diagrams, to specify relationships, including consideration of traversability of a network, such as the possible point-to-point flow of traffic through a set of one-way streets.

Measurement, chance and data

At Level 5 students accurately measure, using rational numbers in fractional and decimal form, the characteristics of length, perimeter, area, surface area, volume, capacity and angle in shapes and solids; and time and temperature. They calculate, using rational and real numbers, formulas for relationships between measurement variables; the area and perimeter of circles, parallelograms and regular polygons; and the surface area and volume (as cross-sectional area $\times$ length dimension) of prisms.

Students evaluate the reasonableness of the accuracy of measurements and give lower and upper bounds for measurement values. They calculate absolute percentage error using the formula

$$\left|\frac{\text{estimated value} - \text{actual value}}{\text{actual value}}\right| \times 100$$

and interpret this in measurement contexts.

Definitions of underlined terms are provided in the Glossary (page 41)

Students demonstrate comprehension of empirical probability as long-run experimental relative frequency, and calculate theoretical probabilities of collections of outcomes in an event space for a random experiment, using symmetry and counting the outcomes in the collections, and comparing them to the total number of possible outcomes in the event space. They use appropriate technology to generate random numbers for simple simulations.

Students organise and present discrete (grouped and ungrouped) and continuous data, using by-hand approaches for small data sets and technology for larger data sets, to represent uni-variate data in dot plots, stem and leaf plots, bar charts and histograms as applicable. They calculate summary statistics that describe measures of centre (mean, median, mode) and spread (range, and mean absolute difference), and make simple inferences based on this data.

Structure

At Level 5 students identify collections of numbers as subsets of natural numbers, integers, rational numbers and real numbers. They specify the relationship between these sets, and subsets of these sets, in terms of set complement, intersection, union and inclusion using Venn diagrams and tree diagrams as appropriate. They list the elements of the power set (the set of all subsets) of a given finite set and comprehend the partial-order relationship between these subsets with respect to inclusion. They specify truth functions for the connectives *and, or, not, implication* and *equivalence*; and the test of the quantifiers *none, some* and *all* in application to statements about elements of a given set. They apply these to the specification of sets defined in terms of one or two attributes, and to Boolean searches using simple combinations of the connectives *not, and, or, if…then…* in databases. They use *ordered* pairs to specify coordinates on graphs and to describe relations between sets.

They apply commutative, associative, and distributive properties in rational and real-number arithmetic and, with respect to subtraction and division, give counter-examples where these properties do not apply. Students identify the identity element and inverse of an arbitrary number for the integers and rational numbers for their respective operations of addition and multiplication. They apply these to the re-arrangement of simple mensuration formulas, and the demonstration of the equivalence between simple algebraic expressions, including equivalences of number expressed in base-exponent form, for example, $2^3 \times 2^5 = 2^8$, $(5^2)^3 = 5^6$ and $(3 \times 4)^2 = 3^2 \times 4^2$. They recognise and apply simple inverse geometric transformations of the plane such as translation, reflection, rotation and dilation.

Students identify a function as a one-to-one correspondence or a many-to-one correspondence between two sets. They represent functions by lists of ordered pairs (tables of values), plots of points on a set of Cartesian axes (graph), and through rules applied to a set of values. They describe and specify the independent and dependent variable of a function and its domain and range. They construct tables of values and draw graphs for functions specified by rules constructed from arithmetic operations, for example, $f(x) = 2x - 4$, $xy = 24$, $y = 2^x$ and $y = x^2 - 3$. They identify rules for such functions from tables of values and use these functions to models for practical situations, in particular, the use of linear functions to model situations where there is a constant rate of change. They solve simple equations, such as $5x + 7 = 23$ and $4x^2 - 3 = 13$, that are amenable to solution by a sequence of inverse operations. They determine the truth values for mathematical expressions, including simple inequalities such as whether the ordered pair (3.5, 6) satisfies the inequaliy $x^2 > 2y$.

Working mathematically

At Level 5 students analyse the reasonableness of points of view and procedures, according to given criteria, and identify limitations and/or constraints in context. They use literal symbols to represent constants, and arbitrary (free) variables in general case arguments, with respect to number, space and structure. They substitute numbers for free variables in equations, inequalities, identities and rules for functions. They give examples of applications of coordinates and functions from historical contexts.

Students develop simple mathematical models for familiar and unfamiliar situations based on the identification of characteristic conditions such as symmetry, invariance and constant rates of change. They apply standard mathematical models and make predictions based on interpolation (working with what is already known) and extrapolation (working beyond what is already known) using known computations and established constructions.

Students use technology for complicated numerical computation, including the construction of tables of values for functions that involve very small and very large numbers. They use technology to implement simple programs for special-purpose algorithms. They transform and manipulate two- and three-dimensional shapes, including projections from three dimensions to two dimensions. They use measuring implements and computer software to construct accurate and detailed representations of shapes and solids. They explain geometric propositions by varying the location of key points and/or lines in a construction.

They use technology, for example, a spreadsheet, graphics calculator or a computer algebra system, to investigate patterns and relations (including equivalence) for simple algebraic expressions.

Level 6

Learning focus

In *Number* students routinely work with natural numbers, integers, rational numbers and irrational numbers as particular types of real numbers in various practical applications and theoretical investigations. They develop familiarity with different representations of these numbers as applicable, including factorials, products of powers of primes, scientific notation, and exact and approximate decimal forms of irrational numbers such as simple surds, the golden ratio, φ, and fractions and multiples of *pi*, π.

Students learn to carry out computations with rational numbers, and rational approximations of irrational real numbers, and to a specified degree of accuracy, to choose and use mental, written or technology-assisted approaches appropriate to the situation. Students carry out computations with simple irrational numbers in their exact form using mental and written methods. They check the accuracy of computations with the decimal forms of real numbers, in terms of the number of decimal places involved and the number of significant figures required.

In *Space* students learn how space is enclosed in two and three dimensions, and how to systematically investigate the properties of boundaries, regions and intersections of boundaries on surfaces with regular shapes such as polygons and circles, prisms and polyhedra. They develop understanding of the orientation of lines in space and the properties of angles formed when these lines intersect.

Students learn to use the concepts of congruency and similarity to compare the size and shape of polygons and apply them to identify and describe shapes that are isometric when they are transformed in the plane by rotation, translation, reflections, and simple combinations of these transformations. They systematically investigate the effects of changing the scale of one characteristic of a geometric shape, for example, length or angle, on the size of related characteristics, for example, area and volume.

Students develop understanding of symmetry as a relationship between parts of a whole in shapes and solids, and explore isometries as a class of transformations of geometric shapes in the plane, the idea of dimensionality in space, and how the dimensions are related to each other. They learn to identify regular and composite shapes that tessellate in two and three dimensions, and use networks to represent relationships such as connectedness.

Students investigate the relationship between position, length and angle using the Pythagorean relationship, trigonometry and the properties of similar triangles and circles.

Definitions of underlined terms are provided in the Glossary (page 41)

In *Measurement, chance and data* students learn to select and use suitable procedures to measure, estimate and calculate length, area, volume, capacity, angle, time, temperature, probability, constant rates of change, density and concentration. In the process they learn to use formulas to calculate lengths, perimeters, areas, angles in shapes, and the surface areas and volumes of solids, and use degrees and radians, as applicable, for units of measurement of angles. They choose real numbers in an appropriate form to describe the size of measurements, and make judgments about errors in measurement. They use and convert units of measurement to suit the purpose of the measurements.

Students learn when to use stem-plots and box-plots to represent data sets and to apply procedures associated with technology to calculate statistics and display uni-variate and bi-variate data sets. They learn to apply probability concepts to aspects of chance and risk in everyday life and represent event spaces that show the nature of events and their probabilities. Students develop understanding of how these representations assist in the computation of the probabilities of compound, independent and dependent events. They apply the concept of mathematical expectation to describe expected gain or loss in games of chance.

In *Structure* students learn to categorise natural, integer, rational and irrational numbers in relation to real numbers. They come to understand the concepts of order, discrete and continuous, and finite and infinite, with respect to subsets of natural, integer, rational and real numbers. They become familiar with the application of algebraic properties, including closure, associative, commutative, identity, inverse and distributive, with respect to natural, integer, rational and real number systems and are able to apply them in the manipulation of mathematical expressions, formulas and equations.

They learn the logical properties of the connectives *and, or, not, implication* and *equivalence*; and the quantifiers *none, some* and *all*, and apply them to the specification of sets for one, two or three attributes, using tree diagrams, Venn diagrams, and Karnaugh maps using complement, intersection and union.

Students work with linear, quadratic and exponential functions, and simple transformations of these functions, their graphs and related algebraic properties. They learn to solve equations of the form $f(x) = k$, where k is a real constant, and simultaneous linear equations using algebraic, numerical and graphical approaches.

In *Working mathematically* students abstract common patterns and structural features from mathematical situations and formulate conjectures, generalisations and arguments in natural language and symbolic form. These conjectures, generalisations and arguments are tested and modified as required and follow formal mathematical arguments for the truth of propositions.

Definitions of underlined terms are provided in the Glossary (page 41)

Students choose, use and develop mathematical models and procedures with attention to assumptions and constraints. They collect relevant data, represent relationships in mathematical terms, and test the suitability of the results obtained in terms of the defining characteristics of the model being used and the features of the context being modelled.

Students engage in investigative tasks and problems set in a wide range of practical, theoretical and historical contexts. Their response to these tasks and problems involves reference to related tasks and previously experienced problems, in particular, to the nature, structure and results obtained in the circumstances.

Students extend the results of their responses to these tasks and problems into more general cases by generalising from one situation to another and changing the initial constraints, or other boundary conditions, of a situation in order to investigate it further. They develop awareness of general features of mathematical structure, the use of logical argument in mathematical discourse and applications of mathematics.

Students use technologies such as geometry software, graphics calculators, spreadsheets and computer algebra systems to assist in the development of mathematical ideas. They carry out relevant computations to support analysis in mathematical inquiry, and to communicate the results of these investigations.

Standards

Number

At Level 6 students comprehend the set of real numbers as the collection that contains natural, integer, rational and irrational numbers, and give examples of each of these. They represent natural numbers as products of powers of primes and rational numbers in both fractional and decimal (terminating and infinite recurring) forms. They express ratios of natural numbers and proportions of these ratios in equivalent fractional, decimal and percentage forms, and use the euclidean division algorithm to find the greatest common divisor of two natural numbers.

Students define and give examples of simple irrational real numbers such as square roots of prime numbers and rational numbers that are not perfect squares, the golden ratio, φ, *pi*, π and simple multiples and fractions of *pi*, π, that correspond to $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{6}$ of a turn around a unit circle. They comprehend that irrational numbers have infinite non-terminating decimal forms and specify rational approximations for these correct to a required decimal place accuracy. They express very small and very large decimal numbers using scientific notation.

Students carry out arithmetic computations involving natural numbers, integers and finite decimals using mental and/or written algorithms (one- or two-digit divisors in the case of division). They use calculators for arithmetic computations involving several operations on natural or rational numbers, and for rational approximations of irrational numbers of any size, including numbers expressed in scientific notation. They carry out exact arithmetic computations involving fractions, square roots of prime numbers and rational numbers that are not perfect squares, and multiples and fractions of *pi*, π, that correspond to simple fractional amounts of turn in a unit circle.

Students use appropriate estimates to evaluate the reasonableness of the results of calculations involving rational and irrational real numbers, and the decimal approximations for them. They carry out computations to a required accuracy in terms of decimal places and/or significant figures.

Space

At Level 6 students represent space enclosed in two- and three-dimensional shapes using lines, curves, polygons and circles in perspective, and isometric drawings, nets and computer-generated images. They describe and compare common and distinctive features of boundaries, surfaces and interiors of common plane and three-dimensional shapes, including cylinders, spheres, cones, prisms and polyhedra.

Students define, identify and use parallel, perpendicular and angle properties of straight lines and properties of circles, relating centre, radius, diameter, chord, arc, semi-circle, circumference, segment, sector and tangent to geometric facts and relationships between these aspects of shapes. They comprehend and use the conditions for shapes to be congruent or similar, and apply isometric and similarity transformations of geometric shapes in the plane. They determine the effect of changing the scale of one characteristic of two- and three-dimensional shapes, for example, side length, area, volume and angle measure, on related characteristics. They identify points that are invariant under a given transformation.

Students characterise the paths (loci) formed by points, lines and shapes as they move in space according to various rules, conditions and/or constraints involving transformation (reflection, rotation and/or translation), distance, latitudes and longitudes, and great circles. They describe the connection between objects/location/events according to defined relationships (networks).

Measurement, chance and data

At Level 6 students estimate and measure characteristics of length, area, volume, capacity, angle, time, temperature, probability, and constant rate of change where one variable is compared to another as a proportion such as the density of substances (mass in relation to volume), concentration of fluids, average speed and measures such as pollution levels in the atmosphere. They choose real numbers in appropriate form to describe the size of measurements, in particular those of very large or small quantities, using scientific notation. They use degrees and radians as units of measurement for angles and convert between units of measurement as appropriate.

Students comprehend the nature of errors of measurement – that error can be compounded by repetition and calculation – and decide on acceptable/tolerable levels of error in a given situation. They interpret and use mensuration formulas for calculating the boundary, surface area and volume of familiar two- and three-dimensional shapes and simple composites of these shapes. They use Pythagoras' theorem and trigonometric ratios (sine, cosine and tangent) to obtain lengths of sides, angles and the area of right-angled triangles.

Students generate data using surveys, experiments, simulations and sampling procedures. They comprehend the difference between a population and a sample, and collect, display and compare uni-variate and bi-variate data sets as samples drawn from populations. They use appropriate summary statistics, obtained by hand (small data sets only) and with technology, to calculate and represent centrality (mode, median and mean), spread (box-and-whisker-plot, inter-quartile range, outliers) and association (by-eye estimation from a scatter plot). They distinguish informally between association and causal relationship in bi-variate data, and make predictions based on a by-eye line of best fit for scatter-plot data with strong association between two variables.

Students estimate probabilities based on published data (experiments, surveys, samples) and assign and justify subjective probabilities, including odds, in familiar situations. They classify events as likely to be dependent or independent. They list event spaces (for combinations of up to three events) by lists, grids, tree diagrams, Venn diagrams and Karnaugh maps, and use these tools to calculate probabilities for complementary, mutually exclusive, and compound events (defined using *and*, *or* and *not*).

Structure

At Level 6 students provide examples of natural numbers, integers, rational numbers and irrational numbers and their properties as subsets of the real-number system. They classify these subsets as discrete or continuous, and finite or infinite, and place any two or more elements of these sets in relative order according to their magnitudes. They apply the algebraic properties (closure, associative, commutative, identity, inverse and distributive) to computation with number, to rearrange formulas, rearrange and simplify algebraic expressions involving real variables, and verify the equivalence or otherwise of algebraic expressions (linear, square, cube, exponent, and reciprocal). They recognise and apply simple compositions of geometric transformations to objects in the plane.

Student express relations between sets using membership, $\in$, complement, $'$, intersection, $\cap$, union, $\cup$, and subset, $\subseteq$, for up to three sets. They represent a universal set as the disjoint union of intersections of up to three sets and their complements, and illustrate this using a tree diagram, Venn diagram or Karnaugh map.

Students identify and represent linear, quadratic and exponential functions by table, rule and graph (all four quadrants of the cartesian coordinate system) with consideration of independent and dependent variables for a given relationship, domain and range. They distinguish between these types of functions (constant first difference, second difference and ratio between consecutive terms of the dependent variable in a table of values; shape of their graphs) and their use and interpretation as models for data where a functional relationship is anticipated.

They recognise and explain the roles of the relevant constants in the relationships $f(x) = ax + c$, $f(x) = a(x + b)^2 + c$ and $f(x) = ca^x$. They solve equations of the form $f(x) = k$, where k is a real constant, and simultaneous linear equations in two variables algebraically, numerically (bisection) and graphically.

Working mathematically

At Level 6 students abstract common and distinctive patterns and structural features from mathematical situations, and formulate conjectures, generalisations and arguments in natural language and symbolic form, for example, the relationship between $f(x)$, $f(y)$ and $f(x + y)$ or $f(xy)$ for reciprocal, square root and exponential functions. They test and modify conjectures, generalisations and arguments (including recently proved, or new and as yet unproved, conjectures) as required, and they follow formal mathematical arguments for the truth of propositions.

Students choose, use and develop mathematical models and procedures, and investigate assumptions and constraints. They collect relevant data, represent relationships in mathematical terms and test the suitability of the results

obtained in terms of the defining characteristics of the model being used and the features of the context being modelled. They routinely make judgments about the reasonableness of computations (calculations, constructions, measurements, inferences, manipulations and deductions) based on the context under consideration.

Students investigate situations and solve problems set in a wide range of practical, theoretical and historical contexts, both within Mathematics and across domains. They consider cases that involve generalising from one situation to another, and changing the initial constraints, or other boundary conditions, of a situation in order to investigate it further.

Students demonstrate awareness of general features of mathematical structure and the use of logical argument in mathematical discourse. They recognise and follow deduction and mathematical induction to establish general results, and distinguish between empirical induction and mathematical induction.

Students use geometry software or graphics calculators to create geometric objects and transform them, taking into account invariance under transformation; graphics calculators, spreadsheets and/or computer algebra system to manipulate and represent data; and computer algebra systems to analyse functions and carry out symbolic manipulation. They select and use technology in various combinations to assist in developing mathematical ideas and carry out relevant computations to support analysis in mathematical inquiry.

The online edition of the Mathematics domain has a comprehensive glossary of mathematical terms with hyperlinks. The following are excerpts from the glossary.

Glossary

Associative

An operation is *associative* if the result of applying the operation to any three elements of an expression is the same regardless of which pair of elements (without changing their order) is combined first.

Addition and multiplication *are* associative on the set of natural numbers, for example:

$4 + (7 + 5) = 4 + 12 = 16$ and

$(4 + 7) + 5 = 11 + 5 = 16$

$2 \times (3 \times 4) = 2 \times 12 = 24$ and

$(2 \times 3) \times 4 = 6 \times 4 = 24$

Subtraction and division are *not* associative on the set of natural numbers, for example:

$10 - (4 - 2) = 10 - 2 = 8$ but

$(10 - 4) - 2 = 6 - 2 = 4$

$24 \div (12 \div 2) = 24 \div 6 = 4$ but

$(24 \div 12) \div 2 = 2 \div 2 = 1$

In general the *associative* laws (properties) for addition and multiplication of real numbers state respectively that *for all* real numbers a, b and c:

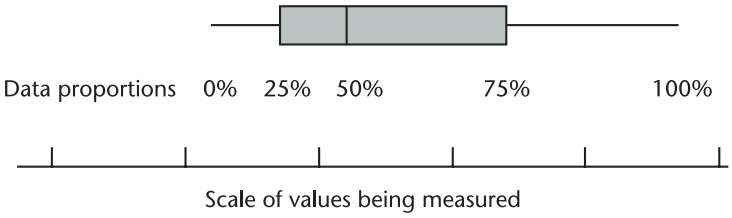
$a + (b + c) = (a + b) + c$ and

$a \times (b \times c) = (a \times b) \times c$

Box-plot

A form of data representation where the ends of a rectangular box are aligned on a numerical scale with a given proportion of a *sample* of uni-variate data. For example, the resting heartbeats of a group of athletes may be measured and a box-plot constructed to correspond to the middle 50% of values. Lines (whiskers) are added to show the lower and upper 25% of the data. The *median* value (the middle value of the sample) is also indicated by a vertical line parallel to the ends of the box:

Box-plot of proportions of the sample



Conjecture

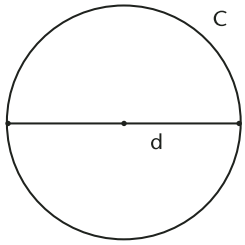
A statement whose truth or otherwise is not yet determined, but is open to further investigation, for example, *Golbach's Conjecture: 'every even natural number greater than 2 can be expressed as a sum of two prime numbers'*. First stated in 1742, the *Golbach conjecture* has not yet been either proven to be true or shown to be false, although many mathematicians intuitively believe that it is true.

Numeral

The *designation* of a number in a given language, for example the number 'three' is designated by the Hindu-Arabic numeral 3, the Roman numeral III, and the Chinese numeral 三

pi

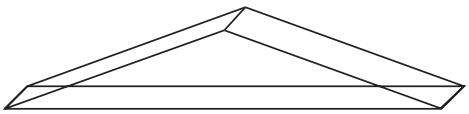
Represented by the symbol π, is the irrational number defined by the ratio of the circumference C of a circle to its diameter, d:



Its approximate value, correct to 2 decimal, places is 3.14, and $\frac{22}{7}$ is a reasonably accurate fraction approximation to π. The decimal expansion for π to 10 significant figures is 3.141592654. The digits in the continued decimal expansion of π do not have any recurring pattern, a property which distinguishes irrational numbers from rational numbers.

Prism

A three dimension shape that has a polygonal cross section, formed by having its edge points translated parallel to a given direction. For example, the following shape is a *triangular prism*:



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